



Guideline for Quantifying the Value of Structural Health Information

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Abstract

1 Introduction

Information gathered by means of measurements or by Structural Health Monitoring (SHM) systems can substantially contribute to an enhanced performance of civil structures and other infrastructure. Such Structural Health Information (SHI) can provide a decision support improving benefit generation and reducing operational costs and risks throughout their life cycle. However, the performance and utility enhancement of technical systems by SHI is often not or not consistently taken into consideration. This prevents a systematic quantification and demonstration of the value and impact of SHI, which may even lead to misdirecting the design and innovation of SHI systems.

The value of SHI can be quantified with the Bayesian decision analysis necessitating the modelling of decision scenarios involving (1) models of the technical system and SHI system life cycle performance and costs, (2) a model representing the decision maker's objectives and (3) decision variables associated to the SHI systems and technical system performance management strategies.

The quantification of the value of SHI provides the basis for a targeted SHI system development, design and innovation to optimize the benefit generation, costs and risks of civil structures and other infrastructures to benefit the sustainable benefit of society.

The Guideline on Quantifying the Value of Structural Health Information aims thus at providing a consistent and comprehensive formulation of the value of SHI quantification encompassing:

- (1) decision scenario modelling with a list of exemplary analyses,
- (2) the description of the decision theoretical foundation,
- (3) the ranking of the SHI strategies on the basis of the value of information,
- (4) the probabilistic modelling of SHI and a summary of SHI strategies and
- (5) a list of approaches to update and adapt the structural performance with SHI.

2 Value of structural health information

The quantification of the value of structural health information (SHI) takes basis in the Bayesian posterior decision analysis (Raiffa and Schlaifer (1961)) in conjunction with utility theory (Von Neumann and Morgenstern (1947)) and its interpretation in the JCSS document on Risk Assessment in Engineering (Faber (2008)). The quantification of the value of SHI requires the definition of a decision scenario consisting of coupled infrastructure and a SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis.

2.1 Decision scenario modelling and analysis

The decision scenario for a value of SHI analysis consists of coupled infrastructure and a SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis. The decision scenario without the temporal dimension may be illustrated with a decision tree (Figure 1, left). Here, the SHI system is described with the information type and its probabilistic outcomes, and the infrastructure performance with the system state models and the actions. The decision tree contains decision nodes to decide upon the utilisation of SHI, the SHI strategy to be utilised and the actions to be performed. The chance nodes stand for the probabilistic models associated to the SHI outcomes and the system states. The utility nodes encompass the decision attributes like benefits, costs and consequences associated to each of the decision tree branches.

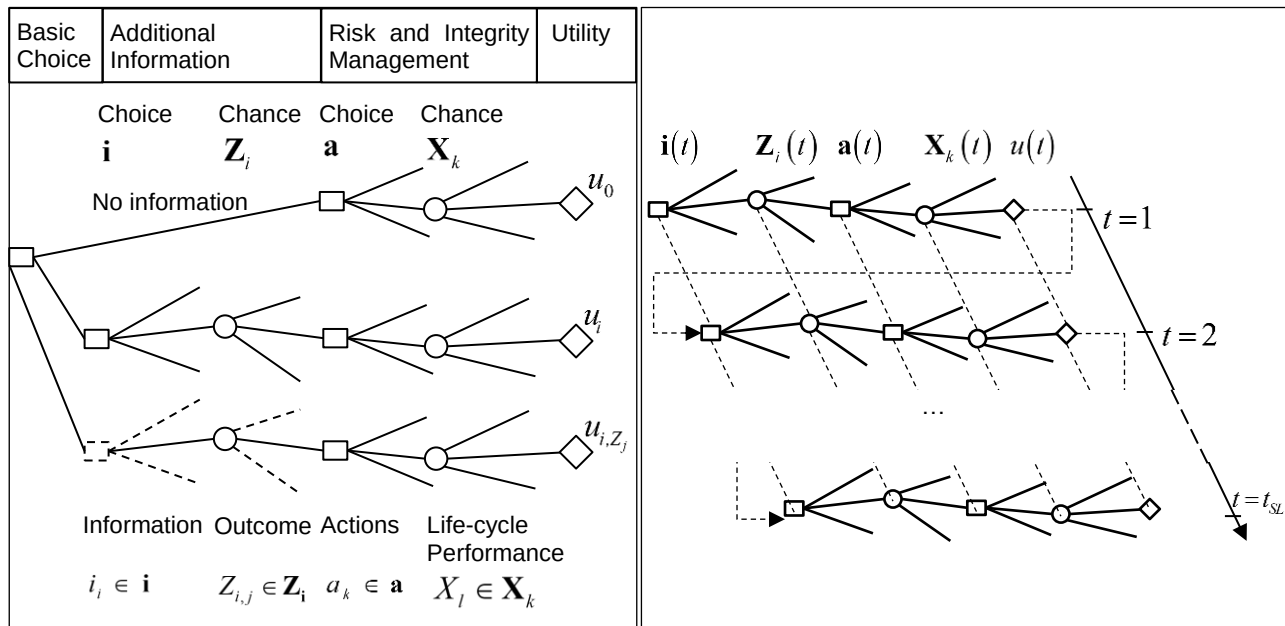


Figure 1: Decision tree for the assessment of the Value of Information containing decision nodes (rectangulars) and chance nodes (circles) and illustration of sequential pre-posterior decision analyses in the time domain

When the decision tree is applied sequentially, it may contain the information gathered in previous time steps and (sub-)periods and common influencing variables. This is symbolized in Figure 1 (right) with the connections between the decision trees in different time steps and with the connections of the decision, chance and utility nodes, respectively.

The temporal modelling of the decision analyses should be allocated and connected to the life cycle phases and the performance of the infrastructure system. The life cycle phases encompass the planning and design, the manufacturing, the construction, the operation and maintenance as well as decommissioning, see Figure 2. The life cycle phase models should be connected to the infrastructure system states namely the intact state, hazard state, constituent and system damage and failure states.

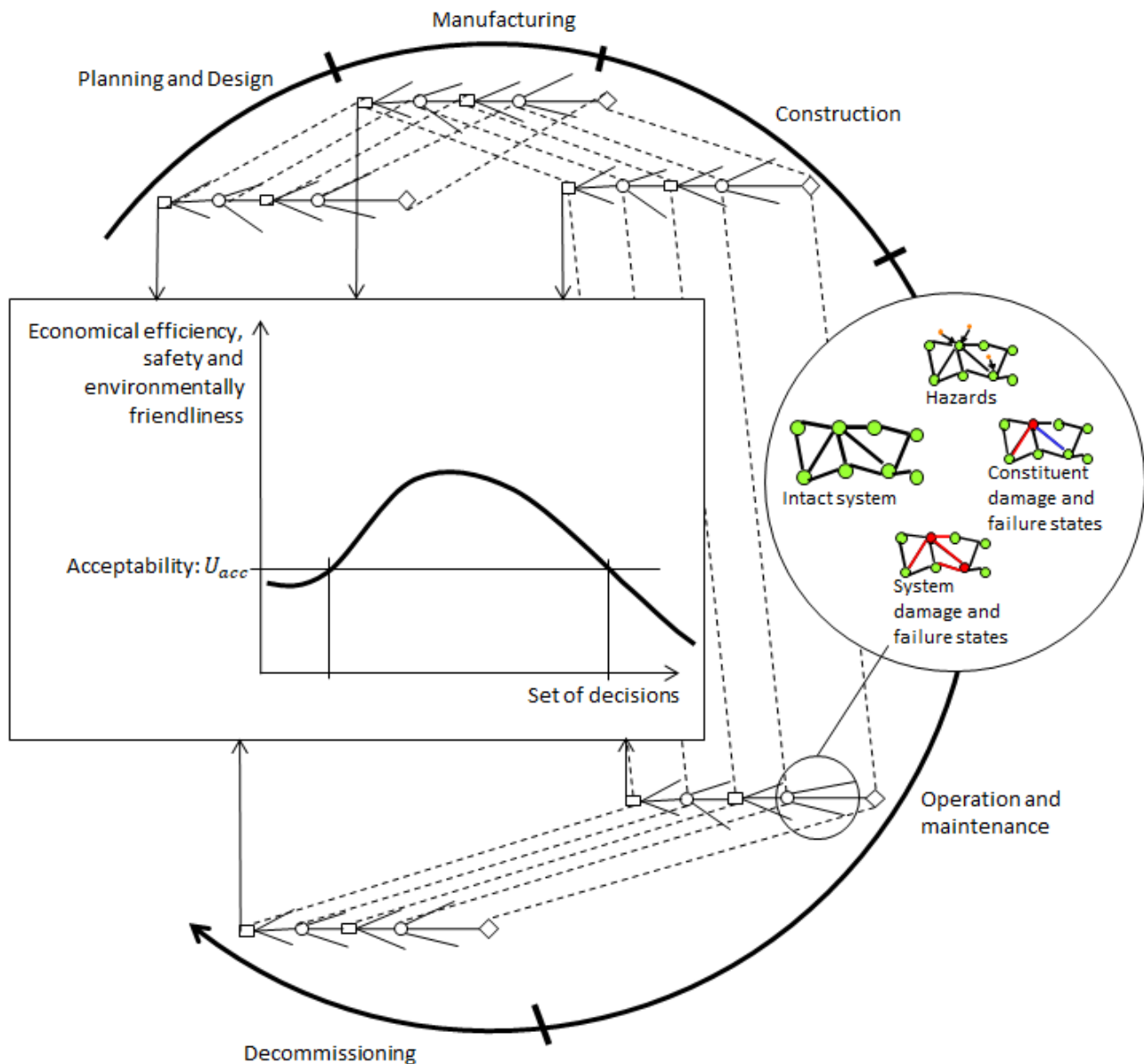


Figure 2: Illustration of a value of SHI analyses throughout the life cycle phases accounting for the intact, hazard, constituent and system damage and failure states (see also Faber, Qin et al. (2017)).

On the basis of the presented framework it becomes obvious that SHI may be of benefit throughout the life cycle of the structure and in any of the constituent and system states. Exemplary decision scenarios may be:

1. Utility, functionality, integrity and risk management in the operation phase of an infrastructure system

In the operation phase, SHI is often utilised for condition assessment, which may serve multiple purposes to plan an efficient utility, functionality, integrity and risk management. Typical purposes include:

- a) Integrity management planning

The integrity management encompassing condition assessment, repair and maintenance actions should be planned and supported with tailored SHI system in the design phase and then continuously adapted to the obtained SHI.



b) Service life extension

For highly utilised structures, an extension of the service life is often desired. Here, SHM information provide means and help for the condition assessment and condition prognosis which form the basis for the optimisation for ensuring the structural integrity beyond the service life.

c) Utilisation modification

The utilisation of structures may change throughout the service life, especially for infrastructures, examples are bridges which sustain a steadily increasing traffic load and e.g. re-powering of wind energy turbines [16]. Here, data about the past utilisation of the structure, the actual condition and performance may provide benefit for the structural integrity management.

d) Functionality enhancement

The utility associated to the functionality of an infrastructure can be supported by the reduction of downtime and closures. SHI to support e.g. the integrity management planning enhancing the functionality can be obtained.

e) Damage progression monitoring

In case that damage has been identified already, monitoring systems may identify eventual trends (i.e. opening of cracks) to monitor eventual increased accelerated deterioration [16]. From one or few monitored locations the damage progression at other locations maybe inferred.

f) Early damage warning for risk mitigation measures

Monitoring may indicate abnormal performance or possible damage of a structure and thus aid as indicators for remedial actions. In this application of SHM, the value of monitoring would relate to the possibility of loss reduction by shutting down the function or reducing the loading of the structure, before human life, the structure and the environment are lost and/or damaged further. Embedded in a maintenance scheme, synergies with the structural integrity management and operation (see point (1)) can be realised.

2. Code and standard calibration - decision support in the design phase

Code calibration for structure types may benefit from SHI when conducted systematically. The acquired information can be used to adapt the design basis in order to spare material and monetary resources while controlling safety, risk and reliability at the desired level. This could be e.g. achieved by reducing the model uncertainties in the design code equations with measuring the relevant magnitudes in the operational structure.

3. Structure prototype development and design by testing – design phase

The production of larger quantities of identical structures can benefit from optimisation processes supported by SHM. A prototype may be equipped with SHM systems in order to attain an optimized structural design before mass production. The SHM data may contain information to reduce uncertainties considered in the design model. Prediction of the response and performance of the prototype and mass produced structure may become more accurate with the use of SHM data. The optimized design may thus lead to an increased life-cycle benefit of the structure [3].



4. SHI system development

SHI systems may be designed and optimized using value of information approaches closely associated to a decision scenario. The SHI system can then be specifically and the technology, number and placement of sensors and the measurement periods can be optimised.

2.2 Systems, utility, consequence and acceptance criteria modelling

The quantification of the value of SHI requires the definition of a decision scenario consisting of coupled - most commonly with a likelihood - infrastructure and a SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis.

The infrastructure system is described with its system states, the associated utilities and physical measures influencing the system states namely actions like e.g. repair and maintenance. These performance models of the infrastructure system should account for the intact state, hazard state, constituent and system damage and failure states including their dependencies and probabilistic characteristics.

The SHI system is described with the information type and its probabilistic outcomes, which can serve as decision variables. The SHI influence to all relevant the infrastructure system states, the associated utilities and actions needs to be explicitly modelled and all SHI system costs throughout the SHI system service life accounted for.

All relevant utilities associated to the entire decision scenario including the system states and the physical measures (actions) and information should be modelled, quantified and discounted with an appropriate discount rate applicable to decision scenario, system boundaries and time horizon. The utilities should encompass generated benefits due to the infrastructure performance and consequences including (i) direct consequences due to constituent and system damage and (ii) indirect consequences, such as (a) loss of life, (b) economic losses, reduction in GDP, etc., (c) social losses and (d) environmental losses.

The system state modelling may build upon a risk analysis and it may be required that risk acceptance criteria have to be accounted for. In this case, standards and regulations applicable to the temporal and spatial scope of the decision analysis must be fulfilled. Present standards and regulations often specify occupational or working risks to individuals and/or risks to society. For societal risks, target probabilities can be derived by utilising the Life Quality index and the marginal live saving principles (see Faber (2008) and ISO 2394 (2015)). This facilitates that the societal preferences and capabilities can be accounted for. The target probabilities can then be compared with the quantified component and system failure probabilities. More details and models can be found in the JCSS document on Risk Assessment in Engineering (Faber (2008)) and the JCSS Probabilistic Model Code.

2.3 Objective functions

The objective function models the decision makers preferences aggregating the system performance, management and utilities according to the decision scenario. The objective function addresses (1) the fundamental decision of considering additional and yet unknown information or not, (2) the identification of an optimal SHI strategy, (3) the identification of optimal actions given SHI and (4) the identification of optimal actions without SHI.

The fundamental decision of considering additional and yet unknown information or not can be based upon maximization of the value of information V . The maximisation of the value of information is calculated maximising the difference between the expected value of the utilities with and without SHI, $U_1(i_i^*, \mathbf{a}^{*,j})$ and $U_0(\mathbf{a}^{*,0})$ respectively, which is performed with the identification of the optimal SHI strategy i_i^* , the SHI outcomes dependent optimal set of actions $\mathbf{a}^{*,j} \in \mathbf{a}$ and the optimal set of actions without SHI, $\mathbf{a}^{*,0} \in \mathbf{a}$.

$$V(i_i^*, \mathbf{a}^{*,j}, a_k^{*,0}) = U_1(i_i^*, \mathbf{a}^{*,j}) - U_0(a_k^{*,0}) \quad \text{with} \quad \{i_i^*, \mathbf{a}^{*,j}, a_k^{*,0}\} = \arg \max_{i_i \in \mathbf{i}, \mathbf{a}^i \in \mathbf{a}} U_1(i_i, \mathbf{a}^i) - \arg \max_{a_k^0 \in \mathbf{a}} U_0(a_k^0) \quad (1)$$



$$\text{s.t. } U_{acc} \leq U_1(i_i^*, \mathbf{a}^{*,j}) \text{ and } U_{acc} \leq U_0(\mathbf{a}^{*,0}) \quad (2)$$

The Value of Information can be normalized in relation to the prior life cycle benefits U_0 resulting in the relative Value of Information, i.e.:

$$\bar{V}(i_i^*, \mathbf{a}^{*,j}, a_k^{*,0}) = \frac{U_1(i_i^*, \mathbf{a}^{*,j}) - U_0(a_k^{*,0})}{U_0(a_k^{*,0})} \text{ s.t. } U_{acc} \leq U_1(i_i^*, \mathbf{a}^{*,j}) \text{ and } U_{acc} \leq U_0(\mathbf{a}^{*,0}) \quad (3)$$

The expected value of the utility with no SHI, $U_0(a_k^{*,0})$, is calculated with the action and chance of the system's life cycle performance $X_l \in \mathbf{X}_k$ dependent expected value of the utility $[u_0(a_k, X_l)]$:

$$U_0(a_k^{*,0}) = \max_{a_k \in \mathbf{a}} E_{X_l} [u_0(a_k^{*,0}, X_l)] \text{ with } \{u_k^{*,0}\} = \arg \max_{a_k \in \mathbf{a}} (E_{X_l} [u_0(a_k, X_l)]) \text{ s.t. } U_{acc} \leq U_0(a_k^{*,0}) \quad (4)$$

For calculating the expected value of the utility $U_1(i_i^*, \mathbf{a}^{*,j})$ in the extensive form, the decision tree is analysed from the right hand side of the decision tree, i.e. from the life cycle performance to the initial starting point: the choice of the information. The expectation over the posterior life cycle performance is calculated by Bayesian updating (operator E_{X_l}'') and the dependency on the life cycle performance is then marginalized with the expectation in regard to the chances of the information (Z_j) outcomes E_{Z_j} :

$$U_1(i_i^*, \mathbf{a}^{*,j}) = E_{Z_j} [E_{X_l}'' [u_i(i_i^*, Z_j, \mathbf{a}^{*,j}, X_l)]] \text{ with } \{i_i^*, \mathbf{a}^{*,j}\} = \arg \max_{i_i \in \mathbf{i}, \mathbf{a}^i \in \mathbf{a}} E_{Z_j} \left[\arg \max_{a_k \in \mathbf{a}} E_{X_l}'' [u_i(i_i, Z_j, a_k, X_l)] \right] \quad (5)$$

$$\text{s.t. } U_{acc} \leq U_1(i_i^*, \mathbf{a}^{*,j}) \quad (6)$$

In the normal form, the decision tree is defined with the decision rules $d_m(Z_{i,j}, a_k) \in \mathbf{d}(Z_i, \mathbf{a})$ linking the actions with the random outcomes of the information acquirement strategy i . The decision tree is analysed starting from the left hand side calculating the expected value in regard to the information outcome for each given life cycle performance with the operator $E_{Z_j|X_l}$. Consecutively, the expectation in regard to the life cycle performance is calculated, i.e. the dependent benefits are marginalized:

$$U_1(i_i^*, \mathbf{d}^*) = E_{X_l} [E_{Z_j|X_l} [u_i(i_i^*, Z_j, \mathbf{d}^*, X_l)]] \text{ with } \{i_i^*, \mathbf{d}^*\} = \arg \max_{i_i \in \mathbf{i}, \mathbf{d}^i \in \mathbf{d}} E_{X_l} [E_{Z_j|X_l} [u_i(i_i, Z_j, \mathbf{d}^i, X_l)]] \quad (7)$$

$$\text{s.t. } U_{acc} \leq U_1(i_i^*, \mathbf{d}^*) \quad (8)$$

For the case that the SHI with an indication has already been acquired, which is denoted with the posterior value of SHI $V | Z_j$, the value of SHI may be assessed retrospectively as the difference of the expected value of the utilities with and without the SHI Z_j , $U_2(a_k^{*,j})$ and $U_0(a_k^{*,0})$. A posterior value of SHI analysis addresses (1) the identification of optimal actions given SHI and (2) the identification of optimal actions without SHI. The value of information is maximised by identifying the optimal actions with and without the SHI, $a_k^{*,j}$ and $a_k^{*,0}$ respectively.

$$V | Z_j = U_2(a_k^{*,j}) - U_0(a_k^{*,0}) \text{ with } a_k^{*,j} = \arg \max_{a_k^i \in \mathbf{a}} U_2(a_k^i) - U_0(a_k^{*,0}) \text{ s.t. } U_{acc} \leq U_2(a_k^{*,j}) \text{ and } U_{acc} \leq U_0(a_k^{*,0}) \quad (9)$$

The expected value of the utility U_2 with SHI the posterior value of SHI $V | Z_j$ is calculated with the posterior expectation and the optimal action $a_k^{*,j}$:



$$U_2(a_k^{*j}) = E_{X_l}'' \left[u_{i,Z_j} (a_k^{*j}, X_l) \right] \text{ with } a_k^{*j} = \arg \max_{a_k \in \mathbf{a}} E_{X_l}'' \left[u_{i,Z_j} (a_k, X_l) \right] \text{ s.t. } U_{acc} \leq U_2(a_k^{*j}) \quad (10)$$

2.4 Documentation and ranking of the value of structural health information

The value of structural health information documentation should encompass a detailed listing of the considered decision scenario, its boundaries, the objective function, the decision alternatives and the expected value of the utilities for each of the decision alternatives.

The full set of decision alternatives referring (1) actions, (2) actions given an SHI outcome (so called decision rules), (3) the ranking of SHI strategies and (4) the ranking of the binary decision alternatives of utilising SHI or not is to be documented.

3 Structural health information

Decisions contributing to an efficient management of infrastructures rely on information conditions. According to Nielsen, Glavind et al. (2018), it can be distinguished between:

- 1) The information is relevant and precise.
- 2) The information is relevant but imprecise.
- 3) The information is irrelevant.
- 4) The information is relevant but incorrect.
- 5) The flow of information is disrupted or delayed.

From this context, it follows that structural health information is information with relevance for the decisions influencing the infrastructure performance and utility. SHI are thus required to be relevant by the temporal period and for the structural functionality, damage and failure events, their consequences and the actions. A SHI model encompasses the information type and content, the probabilistic properties and the costs and consequences. The boundaries of the probabilistic SHI models should be well documented to account for information conditions (above).

3.1 SHI types

The SHI type is classified in the context of the structural system performance models. The temporal (discrete, continuous) and spatial (constituent, system) boundaries and the direct and the indirect measurements of structural performance parameters determine the SHI type. Table 1 lists examples of SHI types.

Table 1: SHI type examples

SHI type	Temporal characteristic	Spatial characteristic	Direct / indirect
Damage detection with a distributed measurement systems and the analysis of static and dynamic behaviour	Continuous or periods of measurements	System or subsystem level	Indirect (indication of damage)
Inspection	Discrete	Constituent	Indirect (indication of damage)
Load testing	Discrete	Constituent, subsystem or system level	Indirect (indication of survival or damage or failure)
Monitoring	Continuous or periods of measurements	Constituent	Direct
Non-destructive testing	Discrete	Constituent	Direct



3.2 Probabilistic characteristics of SHI

The SHI information have probabilistic characteristics originating from (1) the measurement process, (2) the SHI installation and operation and (3) the data analysis. Probabilistic characteristics of the measurement process are caused by the conversion of e.g. electrical or optical signals to structural properties and the process inherent uncertainties relating to the sensor precision, conversion and amplification unit.

During the SHI installation and operation, the probabilistic model should account for (a) the sensor and SHI system installation, (b) the operational conditions, which are not covered by data normalisation and (c) human errors.

The probabilistic models referring to the data analysis should account for (a) the statistical uncertainties due to a limited amount of data and limited measurement periods in relation to the temporal boundaries of the decision scenario, (b) the limited precision of data analysis and data normalisation algorithms and (c) human errors. For consecutive or multiple SHI, the dependency characteristics must be explicitly modelled.

The uncertainties related to the measurement process (see (1)) and the SHI installation and operation (see (2)) can be reduced by calibration of the measurement system and by measuring under specified conditions. The data analysis uncertainties require in most cases explicit probabilistic modelling.

The uncertainties originating from the measurement process, the SHI installation and operation and the data analysis may be modelled based on detection theory, see e.g. Kay (1993) and Gandossi and Annis (2010). The signal distribution of a SHI system may be obtained for the reference state ($d = 0$) and the damaged states with damage d (Figure 3 and Figure 4). With the introduction of a threshold s_t , it can be distinguished between a safe indication Z_1 and damage indication Z_2 .

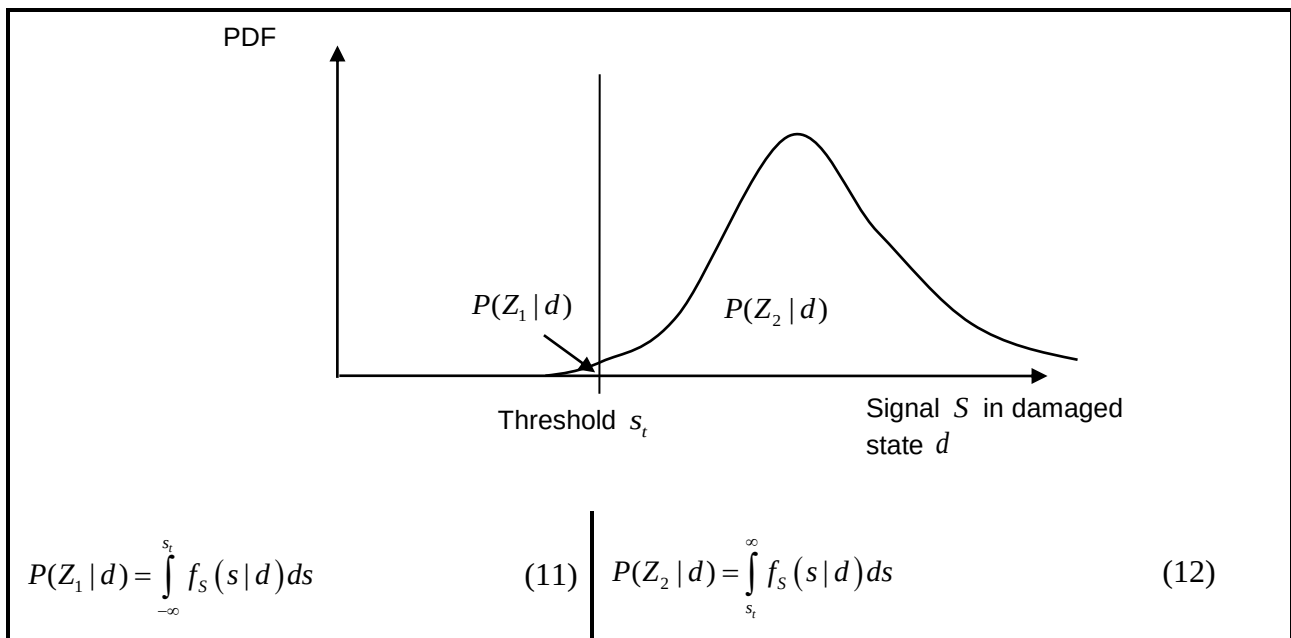


Figure 3: Analytical formulation of the SHM system test outcomes given damage

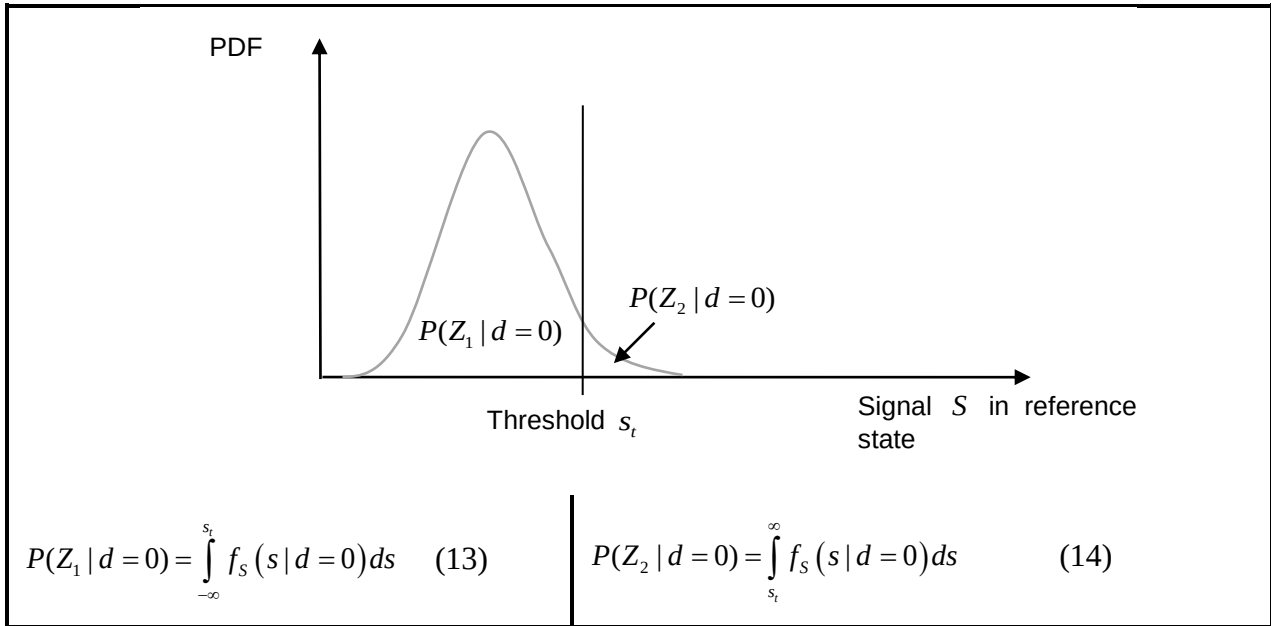


Figure 4: Analytical formulation of the SHM system test outcomes given no damage

The integration of the signal distributions for all damage states (accumulated in the vector $\mathbf{d}$) leads to the probability of damage indication curve $P(Z_2(\mathbf{d}))$, see Figure 5.

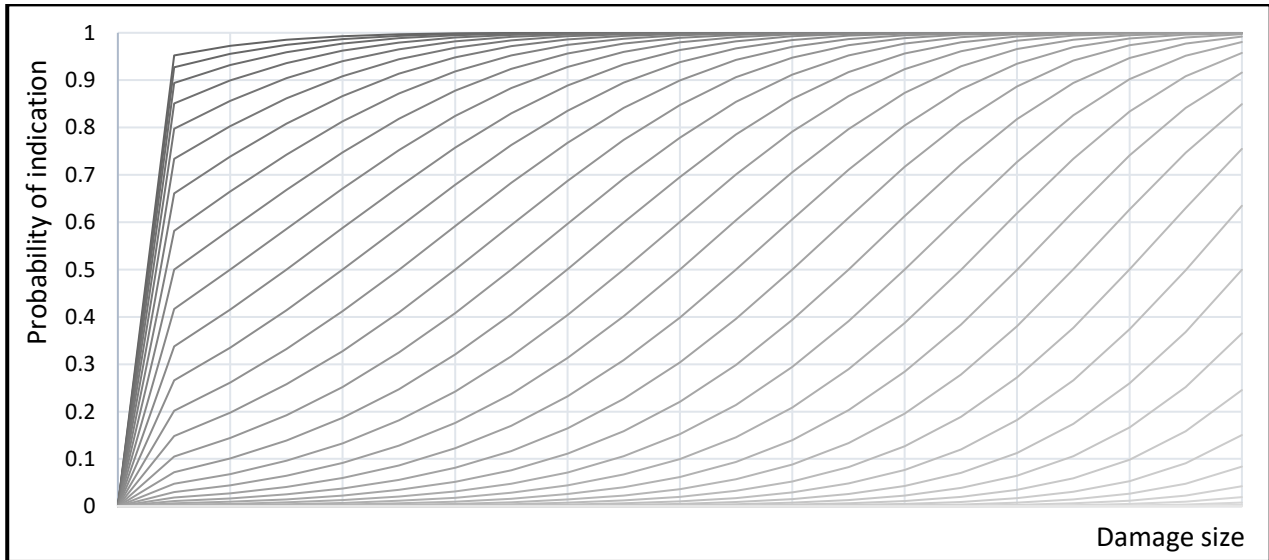


Figure 5: Probability of indication in the damaged states dependent on the threshold (low threshold: dark grey; high threshold: light grey).

The probability of a safe indication given a damaged state can be calculated with $P(Z_1(\mathbf{d})) = 1 - P(Z_2(\mathbf{d}))$. The marginal probabilities of a safe and damage indication curves, which account for the false indications (safe indication but damage state $Z_1(\mathbf{d})$ and damage indication but undamaged state $Z_2(d = 0)$) are calculated with additional consideration of the reference state, i.e.:

$$P(Z_2(d = 0, \mathbf{d})) \text{ and } P(Z_1(d = 0, \mathbf{d})) = 1 - P(Z_2(d = 0, \mathbf{d})) \quad (15)$$



The formulation above can be extended to account for system reference ($\mathbf{d}_s = 0$) and damage states $\mathbf{d}_s$:

$$P(Z_{2,S}(\mathbf{d}_{s,0}, \mathbf{d}_s)) \text{ and } P(Z_{1,S}(\mathbf{d}_{s,0}, \mathbf{d}_s)) = 1 - P(Z_2(\mathbf{d}_{s,0}, \mathbf{d}_s)) \quad (16)$$

The marginal probabilities of a system damage indication are calculated by accounting for all system damage states $\mathbf{d}_s$ and all system reference states $\mathbf{d}_{s,0}$ the resulting n_c -variate signal distribution $f_{n_c, S}$:

$$P(Z_{1,S} | \mathbf{d}_{s,0}, \mathbf{d}_s) = \int_{-\infty}^{s_t} f_{n_c, S}(s | \mathbf{d}_{s,0}, \mathbf{d}_s) ds \text{ and } P(Z_{2,S} | \mathbf{d}_{s,0}, \mathbf{d}_s) = \int_{s_t}^{-\infty} f_{n_c, S}(s | \mathbf{d}_{s,0}, \mathbf{d}_s) ds \quad (17)$$

The high computational demands for the pre-calculation of the multivariate probability of indication curve can be overcome by exploiting specific characteristics of SHI algorithms facilitating a direct calculation of the probability of indication for any system state ({Thöns, 2018 #1600}).

3.3 SHI consequences and costs

The cost and consequences of SHI system investment, installation, operation, renewal and decommissioning require explicit modelling. The service life of the constituent of the SHI and the according replacement intervals within the service life of the infrastructure system should be accounted for. Special attention and explicitness is required for modelling the consequences of SHI false indications.

4 Structural performance adaptation and structural health information modelling

The structural performance adaptation necessitates a consistent modelling of structural health information accounting for their type and probabilistic characteristics in conjunction with the structural system performance models on the basis of Bayesian probability, reliability and decision theory.

4.1 Damage detection information modelling

Damage detection information may be modelled with an indication event such as provided with a damage detection system or an inspection of a system constituent. The modelling of the event requires a limit state function which accounts for the probabilistic information characteristics and is connected to the structural performance states X_l .

The pre-posterior probability (for a value of SHI analysis, i.e. a pre-posterior decision analysis) of the constituent state $X_{1,c}$ with damage detection information for constituents (inspection providing a vector of indications $\mathbf{Z}$) can be calculated by utilising the definition of dependent probabilities.

$$P(X_{1,c} | \mathbf{Z})P(\mathbf{Z}) = P(\mathbf{Z} | X_{1,c})P(X_{1,c}) = P(\mathbf{Z} \cap X_{1,c}) \quad (18)$$

Equ. (18) can be extended for n consecutive damage detection indications $(t_{i,1} \dots t_{i,n})$ throughout the service life and reformulated as a structural reliability problem with the joint distribution of the indications and system states random variables $f_{\mathbf{X}_{X_{1,c}(t)}, \mathbf{X}_{\mathbf{Z}(t_1)}, \dots, \mathbf{X}_{\mathbf{Z}(t_n)}}(\mathbf{x}_{X_{1,c}(t)}, \mathbf{x}_{\mathbf{Z}(t_{i,1})}, \dots, \mathbf{x}_{\mathbf{Z}(t_{i,n})})$ integrated over the limit state function defined domains $\Omega_{X_{1,c}(t)} \cap \Omega_{\mathbf{Z}(t_{i,1})} \cap \dots \cap \Omega_{\mathbf{Z}(t_{i,n})}$:

$$\begin{aligned} & P(X_{1,c}(t) \cap \mathbf{Z}(t_{i,1}) \cap \dots \cap \mathbf{Z}(t_{i,n})) \\ &= \int_{\Omega_{X_{1,c}(t)} \cap \Omega_{\mathbf{Z}(t_{i,1})} \cap \dots \cap \Omega_{\mathbf{Z}(t_{i,n})}} f_{\mathbf{X}_{X_{1,c}(t)}, \mathbf{X}_{\mathbf{Z}(t_1)}, \dots, \mathbf{X}_{\mathbf{Z}(t_n)}}(\mathbf{x}_{X_{1,c}(t)}, \mathbf{x}_{\mathbf{Z}(t_{i,1})}, \dots, \mathbf{x}_{\mathbf{Z}(t_{i,n})}) d\mathbf{x}_{X_{1,c}(t)} d\mathbf{x}_{\mathbf{Z}(t_{i,1})} \dots d\mathbf{x}_{\mathbf{Z}(t_{i,n})} \end{aligned} \quad (19)$$



Once a damage detection indication has been obtained, the posterior probability (for a posterior value of SHI analysis, i.e. a posterior decision analysis) of the constituent state $X_{1,c}$ with the constituent damage detection information Z_j can be calculated by utilising Bayesian updating:

$$P(X_{1,c} | Z_j) = \frac{P(Z_j | X_{1,c})P(X_{1,c})}{P(Z_j)} = \frac{P(Z_j \cap X_{1,c})}{P(Z_j)} \quad (20)$$

For consecutive or multiple indications $(Z_j(t_{i,1}) \dots Z_j(t_{i,n}))$, the posterior probability of the constituent state $X_{1,c}$ can be calculated with extending Equ. (19) and (20), see Equ. (19). The dependencies of consecutive or multiple indications and/or of the individual parameters for describing the indication events need to be explicitly accounted for.

$$P(X_{1,c}(t) | Z_j(t_{i,1}) \dots Z_j(t_{i,n})) = \frac{P(X_{1,c}(t) \cap Z_j(t_{i,1}) \cap \dots \cap Z_j(t_{i,n}))}{P(Z_j(t_{i,1}) \cap \dots \cap Z_j(t_{i,n}))}$$

$$= \frac{\int_{\Omega_{X_{1,c}(t)} \cap \Omega_{Z_j(t_{i,1})} \cap \dots \cap \Omega_{Z_j(t_{i,n})}} f_{\mathbf{x}_{X_{1,c}(t)}, \mathbf{x}_{Z_{j1}(t_{i,1})}, \dots, \mathbf{x}_{Z_{jn}(t_{i,n})}}(\mathbf{x}_{X_{1,c}(t)}, \mathbf{x}_{Z_j(t_{i,1})}, \dots, \mathbf{x}_{Z_j(t_{i,n})}) d\mathbf{x}_{X_{1,c}(t)} d\mathbf{x}_{Z_j(t_{i,1})} \dots d\mathbf{x}_{Z_j(t_{i,n})}}{\int_{\Omega_{Z_j(t_{i,1})} \cap \dots \cap \Omega_{Z_j(t_{i,n})}} f_{\mathbf{x}_{Z_j(t_{i,1})}, \dots, \mathbf{x}_{Z_j(t_{i,n})}}(\mathbf{x}_{Z_j(t_{i,1})}, \dots, \mathbf{x}_{Z_j(t_{i,n})}) d\mathbf{x}_{Z_j(t_{i,1})} \dots d\mathbf{x}_{Z_j(t_{i,n})}} \quad (21)$$

The probability of the constituent state $P(X_{1,c})$ may be written as a structural reliability problem with where the domain Ω_{X_1} is defined with the limit state function:

$$P(X_{1,c}) = \int_{\Omega_{X_{1,c}}} f_{\mathbf{x}_{X_{1,c}}}(\mathbf{x}_{X_{1,c}}) d\mathbf{x}_{X_{1,c}} \quad (22)$$

The constituent limit state function $g_{X_{1,c}}(\mathbf{X}_{X_{1,c}})$ may be exemplarily formulated with the damage dependent resistance $R_c(D_c)$ and loading S_c , which are subjected to the resistance and loading model uncertainties M_{R_c} and M_{S_c} , respectively:

$$X_{1,c} : g_{X_{1,c}}(\mathbf{X}_{X_{1,c}}) = M_{R_c} R_c(D_c) - M_{S_c} S_c \leq 0 \quad (23)$$

The limit state function for the constituent no indication event Z_j is formulated following Hong (1997) with a damage size dependent probability of indication curve $P(Z_j(d=0, \mathbf{d}))$, the damage distribution D_c and a uniformly distributed random variable U :

$$Z_j : g_{Z_j}(\mathbf{X}_{Z_j}) = P(Z_j(D_c)) - U \leq 0 \quad (24)$$

For damage detection information on structural system level, the probability of the system state $X_{1,s}$ can be calculated with the joint distribution of the system state random variables $f_{\mathbf{x}_{X_{1,s}}}(\mathbf{x}_{X_{1,s}})$:



$$P(X_{1,S}) = \int_{\Omega_{X_{1,S}}} f_{X_{1,S}}(\mathbf{x}_{X_{1,S}}) d\mathbf{x}_{X_{1,S}} \quad (25)$$

For the case that the system model can be represented with logical system modelling accounting the redundancy, i.e. statically indeterminacy, of the system, the probability of system failure can be calculated. The structural system is decomposed into n_{j_c} parallel and n_{i_c} series systems (Equ. (24)). The system limit state function can also be formulated considering the redundancy and the brittle or ductile component behaviour with Daniels system modelling.

$$P(X_{1,S}) = P\left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ g_{X_{1,j_c,i_c}}(\mathbf{x}_{X_{1,j_c,i_c}}) = M_{R_{i_c,j_c}} R_{i_c,j_c}(D_{i_c,j_c}) - M_{S_{i_c,j_c}} S_{i_c,j_c} \leq 0 \right\}\right) \quad (26)$$

The limit state function for the constituent no indication event Z_j is readily formulated with a damage size per constituent dependent probability of indication curve $P(Z_{1,S}(\mathbf{d}_{S,0}, \mathbf{d}_S))$ and the vector of constituent

damage distributions $\mathbf{D} = [D_{i_c,j_c} \dots D_{n_{i_c},n_{j_c}}]^T$:

$$Z_{1,S} : g_{Z_{1,S}}(\mathbf{x}_{Z_{1,S}}) = P(Z_{1,S}(\mathbf{D})) - U \leq 0 \quad (27)$$

4.2 Load testing

The adaptation of the system state probabilities with survival information follows the principles as outlined in the previous Section with Equ. (18) to (21) with the constituent and system limit state functions Equ. (23), (25) and (26).

The load testing constituent survival event $Z_{1,c}$ can be modelled building upon the limit state function in Equ. (23) with consideration of the testing load $S_{c,T}$ and the associated model uncertainty $M_{S_{c,T}}$:

$$Z_{1,c} : g_{Z_{1,c}}(\mathbf{x}_{Z_{1,c}}) = M_{R_c} R_c(D_c) - M_{S_{c,T}} S_{c,T} > 0 \quad (28)$$

In specific cases upon justification, the load testing model uncertainty may be neglected and the testing may very controlled having negligible probabilistic characteristics:

$$Z_{1,c} : g_{Z_{1,c}}(\mathbf{x}_{Z_{1,c}}) = M_{R_c} R_c(D_c) - s_{c,T} > 0 \quad (29)$$

The load testing system survival event can be formulated with the proof loading constituent loadings and model uncertainties, $S_{i_c,j_c,T}$ and $M_{S_{i_c,j_c,T}}$, respectively:

$$P(Z_{1,S}) = P\left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ g_{Z_{1,j_c,i_c}}(\mathbf{x}_{Z_{1,j_c,i_c}}) = M_{R_{i_c,j_c}} R_{i_c,j_c}(D_{i_c,j_c}) - M_{S_{i_c,j_c,T}} S_{i_c,j_c,T} \leq 0 \right\}\right) \quad (30)$$

In order to assess the feasibility of the proof load test, the expected gain due to improved information about structural resistance and risks associated with the test (i.e. risk of permanent damage or partial/ full collapse of a structure during the test) needs to be optimised (see e.g. Faber, Val et al. (2000)). It should also be considered that it may be useful to account for the past performance of the structure (i.e. that the structure has survived a certain period of time under service loading).



4.3 Monitoring information modelling

4.3.1 Model uncertainty determination

SHI may be modelled by taking basis in characteristics of model uncertainties. Model uncertainties apply to any parameter of the constituent and system model states and account for model inherent simplifications and assumption. Model uncertainties are determined by means of measurements (see e.g. JCSS (2006), Part 3.09). The process of determining the model uncertainties implies that a built structure constitutes a realization of the model uncertainty. Then, measuring on a structure provides information about the model uncertainty realization subjected to the probabilistic measurement information characteristics.

In the context of a pre-posterior decision analysis, the yet unknown measurement information may be modelled with threshold-truncated distributions of the model uncertainty and the measurement uncertainty. For the case of two thresholds $t_{M_{S,c},1}$ and $t_{M_{S,c},2}$, the probability of the indication may be calculated by the integration of the truncated probability density:

$$\begin{aligned} Z_{1,c} : P(Z_{1,c}) &= \int_{-\infty}^{t_{M_{S,c},1}} f_{M_{S,c}}(m_{S,c}) dm_{S,c} \\ Z_{2,c} : P(Z_{2,c}) &= \int_{t_{M_{S,c},1}}^{t_{M_{S,c},2}} f_{M_{S,c}}(m_{S,c}) dm_{S,c} \\ Z_{3,c} : P(Z_{3,c}) &= \int_{t_{M_{S,c},2}}^{\infty} f_{M_{S,c}}(m_{S,c}) dm_{S,c} \end{aligned} \quad (31)$$

The probability of the constituent state $X_{1,c}$ may be calculated with a limit state function and the threshold-truncated random variable $M_{S,c} [t_{M_{S,c},1}; t_{M_{S,c},2}]$ and the measurement uncertainty M_U (Equ. (30)).

$$g_{X_{1,c}, Z_{2,c}}(X_{1,c}, Z_{2,c}) = M_{R_c}(D_c) - M_{S,c} [t_{M_{S,c},1}; t_{M_{S,c},2}] M_U S_c \leq 0 \quad (32)$$

Already obtained (posterior) measurement information are modelled with the realization $m_{S,c}$ and the measurement uncertainty:

$$g_{X_{1,c}, Z_{2,c}}(X_{1,c}, Z_{2,c}) = M_{R_c}(D_c) - m_{S,c} M_U S_c \leq 0 \quad (33)$$

4.4 Structural performance parameter updating

By means of non-destructive or destructive testing, a likelihood of the resistance $L_{R_c(D_c)}(r_c(d_c))$ may be obtained and used for updating the distribution of $R_c(D_c)$, see Equ. (32). Alternatively, the distribution parameters of $R_c(D_c)$ may be updated.

$$f_{R_c(D_c)}''(r_c(d_c)) = f_{R_c(D_c)}'(r_c(d_c)) \cdot L_{R_c(D_c)}(r_c(d_c)) \cdot \frac{1}{c_n} \quad (34)$$

4.4.1 Equality information modelling

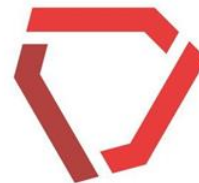
Monitoring information are defined with an equation in contrast to limit state functions, which constitute an inequality. Measurement information can thus not directly be utilised to adapt the system state probabilities (Schall, Gollwitzer et al. (1989)). However, a limit state function representing the measurement information



may be derived with the uncertain measurement M_U and the prior distribution of the measurement value $m(\mathbf{X}_{X_{1,S}})$.

$$Z_{1,S} : g_{Z_{1,S}}(\mathbf{X}_{Z_{1,S}}, M_U) = M_U - m(\mathbf{X}_{X_{1,S}}) \leq 0 \quad (35)$$

With Equ. (35), the likelihood for a measurement outcome given $\mathbf{X}_{X_{1,S}} = \mathbf{x}_{X_{1,S}}$ can be formulated. It can also been shown that a limit state function to solve $P(X_{1,c}(t) \cap \mathbf{Z}(t_{i,1}) \cap \dots \cap \mathbf{Z}(t_{i,n}))$ directly can be derived (see Straub and Papaioannou (2015) and Schneider, Thöns et al. (2017)).



5 Principal example



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